rigid entity with a mass, center of mass, and rotational moment of inertia. (c) The dart is sufficiently long that the atlatl spur, the part of the atlatl which engages the dart, can only apply a horizontal force to the rear end of the dart. (d) The vertical components of force are only included in the torque because their only effect is to cause angular acceleration of the atlatl.

The applied hand force and the reaction force involved in accelerating the dart act upon the center of mass of the hand-atlatl system,

$$M\frac{d^2x}{dt^2}=F(x_h)-M_d\frac{d^2x_d}{dt^2}, \tag{1}$$

where M is the sum of the hand mass, atlatl mass, and any weight added to the atlatl, x is the position of the center of mass of the hand plus atlatl plus any weight added to the atlatl, M_d is the mass of the dart, $F(x_h)$ is the applied horizontal force, and x_d is the horizontal position of the dart. The estimates for the acceleration, a_i, are derived from the second differences of the experimentally obtained position, where i is the time index,

$$a_x(i)=\frac{(x_{i+1}-2x_i+x_{i-1})}{\Delta t^2}, \tag{2}$$

where Δt is the time interval between measurements. The final step is to fit a continuous mathematical function to the experimentally determined force,

$$F(x_h)=900[(x_h+0.05)^{1.5}](0.802-x_h), \tag{3}$$

where x_h is the hand position and the constants were chosen to give the least mean squared error compared with the experimental data. Initially $x_h=0$.

The relation between the angular acceleration, wrist torque, and the hand force and reaction force from accelerating the dart is

$$I_{ha}\frac{d^2}{dt^2}\phi=T(x_h)-F(x_h)L_{ch}\sin(\phi)$$
$$-M_d\frac{d^2}{dt^2}x_dL_{cs}\sin(\phi), \tag{4}$$

where ϕ is the rotation angle of the atlatl relative to horizontal, I_{ha} is the moment of inertia of the hand–atlatl system about the center of mass, $T(x_h)$ is the applied wrist torque, L_{ch} is the distance from center of mass to the hand, and L_{cs} is the distance from the center of mass to the atlatl spur. The continuous function used to fit the experimental torque data is

$$T(x_h)=150(x_h^{3.6})(1.07-x_h). \tag{5}$$

An analysis of the experimental data showed that the maximum force that Baker applied during the throw, 110 N (25 lbs), was about half of the 178 N he could apply statically with a moderate effort to a spring scale. Furthermore, the force started very small at about 8 N, built up gradually to 110 N, and gradually dropped to zero with his hand almost at maximum extension in front and then more rapidly reversed direction as he reached maximum extension at the end of the throw. At maximum forward extension, the reverse directed force was −280 N. The applied wrist torque built up gradually from zero to its maximum value of 18 N m at maximum extension, just before the dart left the atlatl.

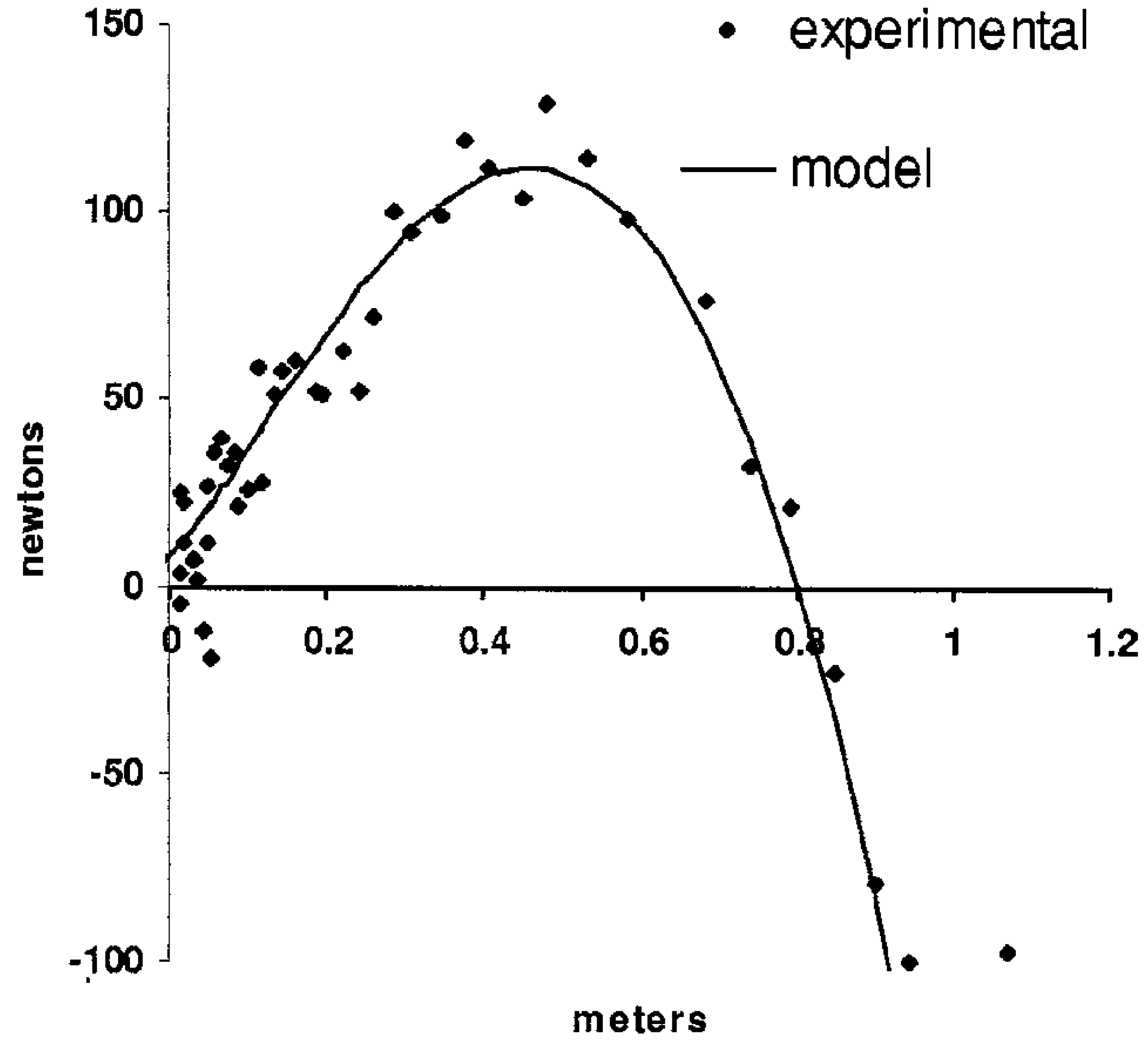

Fig. 3. Measured force versus hand position and the continuous function used to approximate it.

Figures 3 and 4 are the experimentally derived force and torque versus hand position. The noise in the results is primarily due to imperfect spatial resolution in the video digitizer. Also plotted in Figs. 3 and 4 are the results of the models used to represent the force and torque versus hand position.

V. GOING FROM FORCE TO VELOCITY

The hand force was derived from the acceleration data and then, knowing the applied force, the torque was derived from the angular acceleration in a straightforward manner. The reverse process, deriving the position and velocity from the applied force and torque, is more difficult. In order to do this it is instructive to write down the Lagrangian for this system. An initial attempt used x, the horizontal position of the center of mass, and ϕ, the angle of the atlatl, as coordinates. The kinetic energy, expressed in terms of those coordinates and their time derivatives, is

$$KE_1=\tfrac{1}{2}\{M_{ha}v_x^2+I_{ha}\omega^2+M_d[v_x+L_{cs}\omega\sin(\phi)]^2\}. \tag{6}$$

Equation (6) contains a term, $v_x\omega$, that complicates the analysis. A simple expedient eliminates this term. Figure 5 shows a massless spring with spring constant k inserted between the atlatl spur and the proximal end of the dart. In

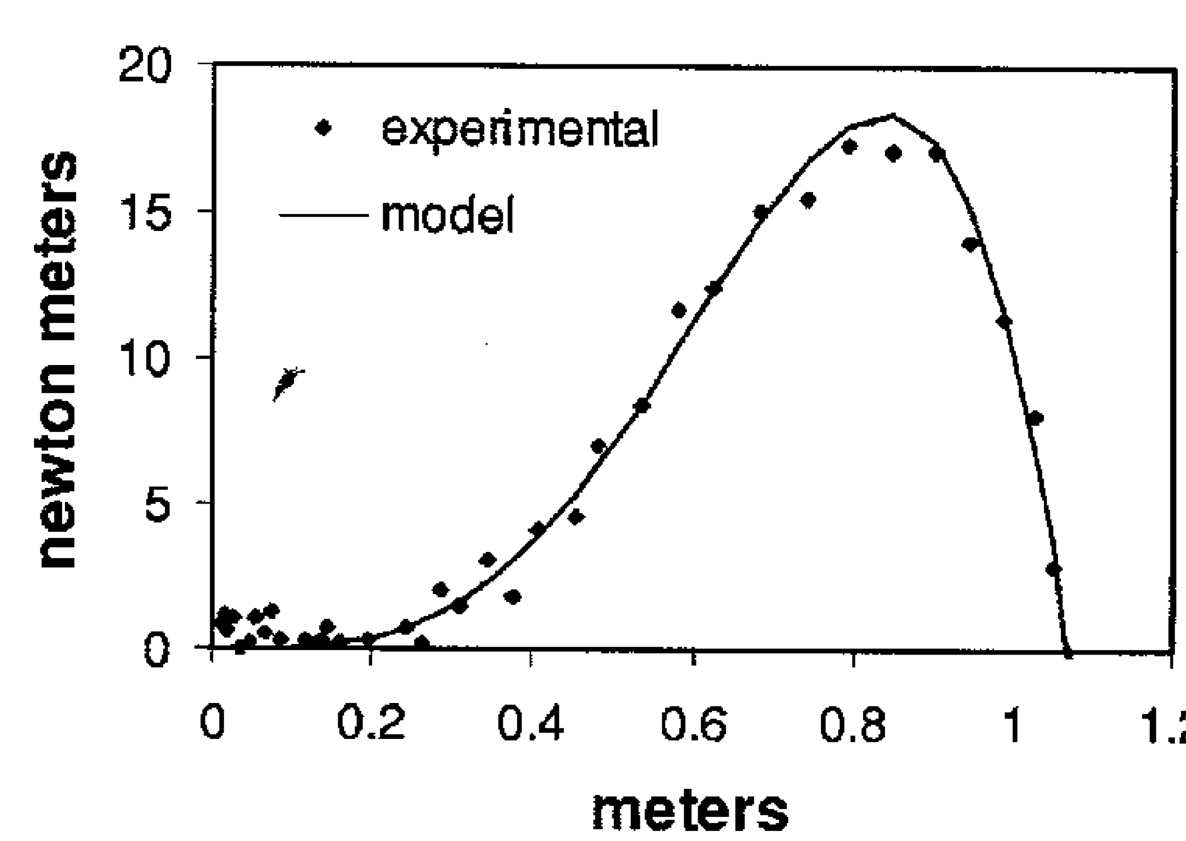

Fig. 4. Measured torque versus hand position and the continuous function used to approximate it.